MODELING AND FORECAST OF SOCIO-ECONOMIC PROCESSES

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Computer Analysis of Qualitative Features in the Formation of the Socio-Ideological Structure of Society*

Yurii N. GAVRILETS
Central Economics and Mathematics Institute RAS
Moscow, Russian Federation, 47, Nakhimovsky Avenue, 117418
E-mail: yurkag@mail.ru

Iraida V. TARAKANOVA
Central Economics and Mathematics Institute RAS
Moscow, Russian Federation, 47, Nakhimovsky Avenue, 117418
E-mail: Itar4@mail.ru

Abstract. The goal of the present paper is to develop a special methodology for analyzing and forecasting mass social interactions; the methodology can be used in various studies of how social positions and opinions of the population are formed. The paper proposes a technique of mathematical modeling of mass social interactions in the context of a comprehensive interplay of social contacts, under the impact of information channels and the external environment. The model is based on substantial ideas about cause-effect relationships between the size of social groups and their changes as a result of mutual transitions, the ideas about how the views of other participants and various media influence socio-psychological attitudes, and about possible external impact on the effectiveness of propaganda. In contrast to commonly used differential equations focusing on the analysis of the stationary state, we propose to analyze the group size dynamics with the use of simple modifications of Markov chains when participants do not move from one group to another uniformly, but are distributed through several time periods, and the probabilities

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of such a transition depend on the current state. For this purpose, computer programs that express the iterative procedures of Markov chains with additional intermediate states are used. An important feature that distinguishes our approach from other models consists in the reliance on a sociological theory that requires social behavior be considered as dependent on variables such as socio-psychological attitudes of members of social groups. Our work also takes into account the factors that determine the attitudes, namely social contacts between people and information channels of different types available to the participants. With the help of simulation calculations, we show how this attitude may affect the inter-group transitions in the formation of a socio-ideological structure of society. In general, these models are a kind of situational stand where we can study the features of formation of the ideological structure of society. If statistical or expert data are available, then the model we have developed can be used to analyze regional socio-political issues.

Key words: attitude, ideological structure of society, social groups, social contacts, the media, dynamics, probability of inter-group transitions.

1. The importance of information in shaping human behavior

The enormous complexity of the modern world, information flows that cover all its parts, computers and electronic equipment that have become necessary management tools – all this led to the emergence of concepts such as "digital economy", "electronic government", etc. The need for effective managerial decision-making on all socio-economic issues allows us to speak about "digital sociology" in a broad sense.

Information generated in a society becomes to a certain extent a demiurge of this society because it increasingly affects people's opinions about the reality around them and forms the preferences of social groups. Regular attacks of the U.S. media on their President, debates in social networks and about social networks, endless controversial speeches and comments of various politicians and experts, demonstrations convened through the Internet, and clashes at rallies – all this turns society in a boiling sea of living people and information flows.

In this regard, the researchers who study socio-economic and political reality face a task to understand the attitudes and causal relationships in the interaction of information and social groups [1, p. 207-214].

The influence of the media, the Internet, etc. on public opinion and on socio-political behavior is considered in many works, both in Russia and abroad. Numerous works by Western and Russian authors mainly focused on simulating the spread of rumors as infection [2, p. 50-54; 3, p. 457-470; 4; 5], analyze the impact of advertising on consumer behavior, emergence of gregarious behavior in people, lust for power [6; 7, p. 58-64], formation of protest groups and preparation of revolutions [8, p. 440-460; 9, p. 76-188; 10, p. 384-434]. Several works [11; 12] contain harsh criticism of the mass media that engage in brainwashing people under the guise of disseminating democracy; the works [13;14] address general problems of modern civilization in this aspect. A large group of Russian researchers from the Institute of Applied Mathematics under the Russian Academy of Sciences and the Department of Sociology at Lomonosov Moscow State University is working on the models of rumor spreading and has developed a mathematical model in the form of differential equations for describing information warfare in society [15; 16, p. 65-74; 17, p. 29]. In recent years, researchers at the Central Economics and Mathematics Institute of RAS analyzed models of ideological confrontation in society and conducted statistical studies of the drivers of protest movement in Russia [18, p. 45-66].

In October 2016, Lomonosov Moscow State University hosted the all-Russian research-to-practice conference “Situation centers 2016”; its participants discussed the issues of creating a system of distributed situation centers in Russia operating under unified regulations. Some research findings of scientists from RAS and the Higher School were also presented at the conference. Eminent scholars and practitioners point out that such centers established in different regions of the country should be able to analyze the socio-economic and socio-political situation and provide the governing bodies of different levels with science-based solutions. Of course, elaborating such solutions should be based on the processing of large amounts of diverse information that reflects the situation; this involves arranging it in the form of some mathematical model allowing for its computer processing.

2. The goal of the study

The goal of this article is to develop a special methodology for analyzing and forecasting *mass* social interactions, a methodology that can be useful for the functioning of situation centers. It is dedicated to computer simulation of the correlations that naturally emerge in the processes of interaction at the “political and ideological field” between social groups, the media, and the external environment. The work involves the possibility of using special statistical information, but when we carried out computational experiments and analyzed their results, we used conditional data. At a purely speculative, qualitative level, it is impossible to trace the consequences of actions of all the participants of social interaction and detect how their multiple characteristics and properties will change. Therefore, the use of computers, even when dealing with conditional data, allows us to identify the role of correlations between such parameters as the availability of information channels, strength of their impact, people’s inclination to imitate, random factor, etc.

As for the nature of its mathematical tools and modeling technique, the present work has some common aspects with the approaches described in [16, p. 65-74; 17, p. 29], etc., because these works, like ours, analyze the properties of models of virtual reality.

Mathematically elegant models in the form of differential equations designed by the authors mentioned above are useful, because they help obtain an explicit expression of the steady-state condition and analyze its dependence on the model parameters. However, for practical calculations, for example, forecast calculations, we consider it more convenient to use difference equations that are inherently close to the cause and Markovian dependencies. Especially for practical use of the model, it is necessary to conduct statistical estimation of parameters not only for the ultimate limit states but also at other time intervals. For this reason, we introduce difference ratios from the very beginning, and indicate the possibilities of their reflecting individual differences of the participants by introducing intermediate states. After that, computational experiments with virtual reality are conducted that help consider the properties of the society and provide an opportunity to see various scenarios of social change as a result of complex interactions between social groups, the mass media and the external environment.

It is necessary to mention that *qualitative conclusions* can be made already at the early stages of the process on the basis of quantitative calculations with the use of computer technology. In the presence of quantitative estimates of model parameters, it becomes possible to analyze real socio-political situations and to solve specific problems of forecast and management.

Let us point out two more features of our approach compared to the above-mentioned mathematical approach to the overall description of the processes of information dissemination. First, we follow the substantive sociological ideas about the dependence of behavior upon an individual's *attitude* that can be measured quantitatively and is affected both by direct social contacts and the media. Second, we examine the *polarity of interests* of major population groups.

3. Socio-ethical attitudes of social groups

We consider the following assumptions about the properties of reality, which are then expressed in mathematical language:

The community under consideration (this community may include young people, population of the region, the electorate on the eve of an election, etc.) consists of several social groups, and each of them has its own type of attitude toward reality and toward possible behavior.

People's attitude toward reality, their subjective assessments (usually positive or negative) and inclination toward something are latent characteristics manifested in certain statements or behavior. We call it an attitude characterizing the readiness for a certain behavior, and this attitude can be measured by certain scales [19]. Examples of attitudes include assessments of programs and slogans of political parties, opinions concerning free sex and moral rigor, tolerance, religious faith and atheism, etc.

Sociologists have found that the attitudes of group members can change either as a result of social contacts (the effect of imitation) or under the influence of information received via media channels, advertising, etc. In turn, the attitudes of group members influence the behavior, in particular, the probability of making certain decisions. As a result of the behavior that people implement, there emerge different groups of conventionally the same type; this defines the social structure of society in the context of appropriate attitudes and behavior.

4. Key variables and parameters of the model

The model considers three major groups (two active and one neutral) and a few "intermediate" groups, whose members quit the major groups and do not participate in social contacts for some time.

In model calculations the fixed parameters and variables for the moment of time t are designated as follows:

x_t – the number of participants of the first active group;

z_t – the number of participants of the second active group;

y_t – the number of participants of the neutral ("passive") group;

$e1_t$, $e2_t$ – quantitative expression of the attitudes to support the goals of the first active and second active groups, expressed on a certain numerical scale;

$E1$, $E2$ – quantitative expression of the values of attitudes "imposed" by the media (typically, the amount of negative and positive information toward the behavior and goals of active groups), measured on the same scale as $e1$, $e2$;

ρ, π – probabilities of autonomous reduction in the size of the first and second active groups at the moment of time t,

$p \cdot \Phi(e)$, $r \cdot \Phi(e)$ – the product of parameters p, r and the monotone function of a participant's attitude (in our case, the so-called error function integral), denoting the probability of transition from the "passive" group to the first or second active groups under the influence of acquired attitude e,

$A1$, $A2$ – parameters indicating the strength of influence of deviation from the imposed standard on the alteration of the current value of the attitude, *B1, B2* – parameters indicating the strength of influence of ideas of a participant concerning the numerical ratio between active groups on the alteration of the current value of the attitude.

5. Socio-ethical structure of society and the change of this structure

The socio-ethical structure of society is determined by the ratio of group sizes: *x, y, z,* when the participants of the first (*x*) and second (*z*) active groups identify themselves as such, but the participants of the passive group (*y*) do not identify themselves as being part of any of the active groups. Examples of socio-political attitude can be found in the intention to vote for a particular party, the degree of confidence in an electoral candidate or public figure, the assessment of their certain qualities, etc.; all these aspects are measured by sociologists.

The value of the attitude is changed under the influence of two processes of communication: under the influence of possible contacts with representatives of the active groups, and under the influence of information received via the corresponding channel. According to the assumption of the models, the information of the channel "recommends" each recipient a specific value of "standard" attitude (*E1* or *E2*), this value expresses a negative or positive tone toward the subject of agitation, which can be measured on a numerical scale.

The difference between the imposed standard and the current value of the attitude *E-e* creates a psychological discomfort which under the additional influence of social contacts with other participants may change in either direction. According to the simplest testable hypothesis [20, p. 110-134], a possible change in attitude caused by direct influence of the imposed standard is in proportion to the magnitude of this difference:

$$\Delta = A \cdot (E - e).$$

We can also assume that the change is nonlinear, it decreases as it gets closer to the standard:

$$\Delta = A \cdot (cnorm(E - e) - 0.5).$$

The function *cnorm(·)* represents the so-called error function integral, it increases monotonically from 0 to 1 (here we follow the approach of a renowned scientist [15]).

Figure 1 shows that if the value of the attitude $u < E1 = 5$, then the magnitude of its change is positive, when $u > E1$, then the magnitude is negative. It is usually considered that the influence on the change in the attitude caused by possible social contacts depends on the frequency of meetings with supporters of both positions. The probability of such contacts can be simulated considering it proportionate either to the predominance of the share of one or another position among the activists, or to this share in relation to the size of the entire neutral group. It is obvious that the type of statistical relationship should be identified by the data of objective observation. Then the magnitude of change can be represented, for example, as follows:

$$A \cdot (E - e) + B \cdot \frac{x - z}{x + z} \cdot y \ ,$$

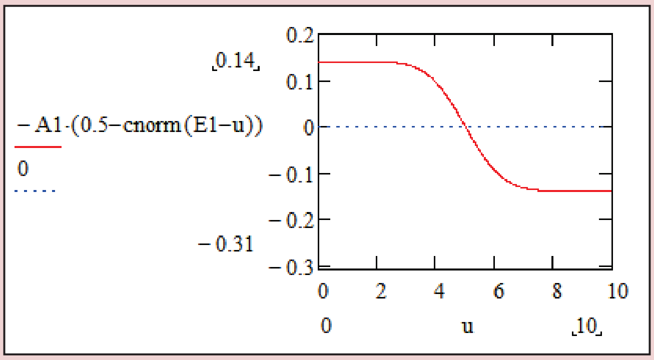
or like this:

$$A \cdot (cnorm(E - e) - 0.5) + B \cdot \frac{x - z}{x + z} \cdot y \ , \quad (1)$$

where the coefficients *A* and *B* compare the effect on the current attitude of the difference from the standard *E* and the numerical superiority *(x-z)* of a particular group. If the sum is positive, then the value of the attitude will increase; if the sum is negative, the value will decrease.

Let us now express the meaning of the basic model with the help of the formulas for a simple equilibrium macro model, when the

Figure 1. Nonlinear dependence of change in the attitude



Compiled with the use of our own calculations.

requirements of equality to zero in the change in the size of the groups *x, y, z*, and changes in the attitudes *E1* and *E2* are fulfilled.

Equilibrium equation (*Ia model*):

1. $-x\cdot\rho + r\cdot cnorm(e1)\cdot y = 0$
2. $-r\cdot cnorm(e1)\cdot y - p\cdot cnorm(e2)\cdot y + \rho\cdot x + \pi\cdot z = 0$
3. $-\pi\cdot z + p\cdot cnorm(e2)\cdot y = 0$
4. $A1\cdot(E1 - e1) + B1\cdot\frac{x-z}{x+z}\cdot y = 0$
5. $A2\cdot(E2 - e2) + B2\cdot\frac{z-x}{x+z}\cdot y = 0$
6. $x + y + z = N$, где N – общее число участников.

The first equation shows that in the equilibrium the number of those who left (ρ – the share of those who left the group) the first active group is fully compensated by the share of those participants of the neutral group who received the information from the sources of the first type and received the attitude *e1* under the influence of which they left the neutral group with the probability $r\cdot cnorm(e1)$. The third equation is similar to the first one and describes the situation with the equilibrium of *z*-members. The second equation indicates the equality between the number of those who came from the active groups and the number of those who left to join the active groups.

Thus, participants from the active groups move autonomously to the neutral group with the known probabilities (ρ, π). Participants move to the active groups from the neutral group with the probabilities $r\cdot cnorm(e1)$ and $p\cdot cnorm(e2)$. The fourth and fifth equations show how the influence of information *E1, E2* from the mass media on the current attitude *e1, e2* (influence coefficients *A1, A2*) correlates with the information obtained through intergroup contacts (influence coefficients *B1, B2*). The values of parameters at which the equilibrium of this model were calculated are as follows:

$A1$=0.28, $A2$=0.28, $B1$=0.05,
$B2$=0.045, $E1$=6,

$E2$=6, r=0.01, p=0.015,

ρ=0.015, π=0.015, N=1000.

The solution of these equilibrium equations is represented by the following values of the variables:

x=181.531, y=106.762, z=711.717,
$e1$=0, $e2$=6.

Due to the differences in the parameters B2>*B1* and p>*r*, indicating a greater inclination toward imitation and a greater impact of the attitude to transition, the z-group receives a distinct advantage: $z>x$.

6. Models of the dynamics of the structure of society

The dynamics of the sizes for this simplest case can be represented by a system of difference equations in which we add logical conditions that specify the interval for the values of the attitudes. Time in the models is discrete and runs through the values from $t_0=0$ to the required number of iterations. At all the calculations conducted, the original data (parameters) were set artificially and expertly, taking into consideration previous studies [20, p. 110-134; 21, pp. 107-118]. Additionally, several thousand alternative calculations were conducted to check structural stability of the models.

Speaking about the specified parameters of the model, it is necessary to emphasize one fact that is not always given sufficient attention when simulating socio-economic processes. We are talking about the seemingly auxiliary parameters with the help of which we move from differential equations to difference equations: about the parameters h_k that specify the step size or the rate of change of the indicators of an iterative process.

No doubt, differential equations provide (due to their smallness) the closeness of solutions of difference equations and differential equations. But their quantitative relationship to each other determines the stability of the stationary state. And in the case of the models of "Markovian" nature, they determine the rate at which the simulated parameters actually change. In fact, they are equal parameters of the model and, just as others, should be evaluated statistically according to observations, as we did in other works. With respect to the models under consideration, we note that the values of the h-coefficients for the variables x, y, z can be considered identical due to the homogeneity of the indicators of size. The same is to some extent true for the h-coefficients of the variables $e1$ and $e2$. As for their numerical values, it depends on the general assumptions of the model. In a particular simulation, we can specify which propaganda has the influence that is adopted faster, and which – slower.

The simplest dynamic *model 1b* has the same parameters and initial values of the variables: $x_0=50$, $y_0=900$, $z_0=50$; the rate of change $h_0=0.1$, $h_1=0.1$, $h_2=0.1$, $h_3=0.015$, $h_4=0.015$:

$$x_{t+1} = x_t + h_0 \cdot [-x_t \cdot \rho + r \cdot cnorm(e1_t) \cdot y_t]$$

$$y_{t+1} = y_t + h_1 \cdot [-r \cdot cnorm(e1_t) \cdot y_t - p \cdot cnorm(e2_t) \cdot y_t + \rho \cdot x_t + \pi \cdot z_t]$$

$$z_{t+1} = z_t + h_2 \cdot [-\pi \cdot z_t + p \cdot cnorm(e2_t) \cdot y_t]$$

$$e1_{t+1} = if\left[e1_t \le 0, 0, if\left[e1_t \ge E1, E1, e1_t + h_3\left[A1 \cdot (E1 - e1_t) + B1 \cdot y_t \cdot \frac{x_t - z_t}{x_t + z_t}\right]\right]\right]$$

$$e2_{t+1} = if\left[e2_t \le 0, 0, if\left[e2_t \ge E2, E2, e2_t + h_4\left[A2 \cdot (E2 - e2_t) + B2 \cdot y_t \cdot \frac{z_t - x_t}{x_t + z_t}\right]\right]\right]$$

When simulating the changes in attitudes, without losing generality, it is assumed that scale values are in the range from 0 to 6, and that it is possible to choose the scales with other ranges depending on specific cases. In order to avoid obtaining the values of attitudes beyond zero and *E1, E2* in the calculations, we introduce special logical conditions that provide the necessary restrictions in the last two equations of the system.

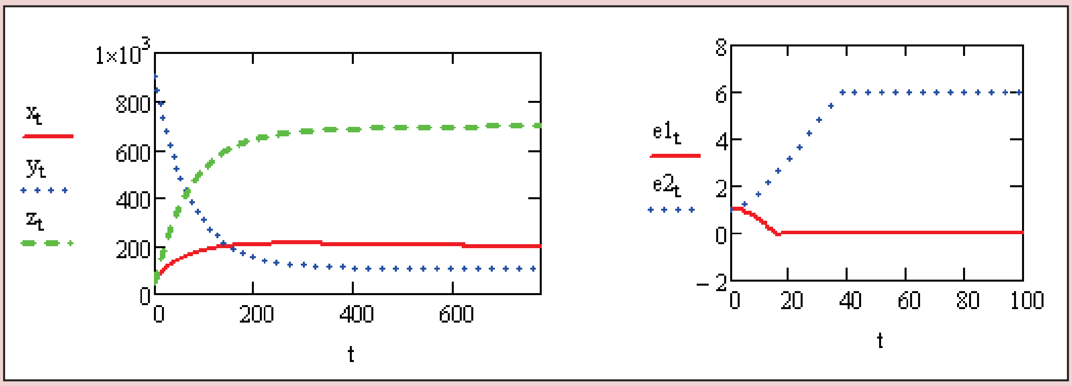
As a result of the calculations, the stationary solution of this system (the state at $t = 500$) coincides with the current balance. We note that the total change in all the sizes is N, which means the total size of all the groups remains unchanged with the passage of time. The trajectories of behavior starting from the initial state depend on the values of the model parameters.

The graph in *Figure 2* shows that already at the 400-th iteration the process stabilizes and coincides with the equilibrium.

Now it is necessary to point out that simulating the dynamics of the attitudes with the use of successive iterations according to the type of standard Markov chains, like in *Model 1b*, (as well as with a standard solution of differential equations using difference schemes) has a significant drawback. Since the probability of transition of an individual member of the group (group's share) depends only on the group itself, it will be the same for all its members regardless of when and how the member got into this group, i.e. all group members are similar and they make transitions simultaneously.

Admittedly, we do not consider the transitions of Markov chains, but rather those of "Markov-type" chains (introducing the rate of change expressed by the *h*-coefficients), because only a small part of the group change their state in one phase. Still, the lack of general "Markov behavior" weakens the model; and to reduce this defect we introduce some number of "intermediate" groups – for the moments of transitions – whose members have become different. As a result, in order to become an "equal" member of a particular group, the member that moves from one group to another must pass through several stages. This technique, of course, increases the number of variables and the dimensionality of the system, but at the same time brings the model closer to reality.

Figure 2. Trajectories of the dynamics of the model variables



Compiled with the use of our own calculations.

Next, we design and examine this more plausible model, which is like a “small” version of the so-called “agent-based” models [22], in which the participants are represented by individuals.

Additionally, we introduce a more specific assumption about the dependence of the number of those who move from one group to another on the prevalence and accessibility of information channels. This fact is reflected in the model by the probability of connecting to the channels (*q1* and *q2*). In addition to the previous model (except for the introduction of these probabilities) we will reflect the influence of the frequency of social contacts on the transition.

A neutral participant, depending on the prevalence of the media of the first or second channel with the probability *q1* or *q2* joins a subgroup of the carriers of the respective attitude (*e1* or *e2*) who assimilate the information of the orientation *E1* or *E2*. Modification of the current attitude is described by the above formula (1). After that, as a result of communication with active members and under the influence of the adopted attitude (*e1* or *e2*) the transition itself happens with a probability that depends on both results of communication. Those who came under the influence of the *x*-ideology make up $q1 \cdot y$ participants. The probability of “going further” depends on the number of its supporters and on the information-psychological pressure on the part of the media, which leads to an increase in the number of supporters of the *x*-ideology by the value $q1 \cdot \left(r \cdot \frac{x}{x+z} + G1 \cdot cnorm(e1) \right) \cdot y$. Those who came under the influence of the *z*-ideology form $q2 \cdot y$ participants. This leads to an increase in the number of supporters of the *z*-ideology by the value $q1 \cdot \left(r \cdot \frac{x}{x+z} + G1 \cdot cnorm(e1) \right) \cdot y$. Naturally, both fractions in these formulas denoting the ratio between the sizes of the groups in reality cannot be known to a particular participant. However, if they reflect the actual and changing proportion between the *types* of people in a society [23; 24], then, according to the principles of social psychology [25] in modern information society, the influence of this fact on the behavior should not be questioned, and modern statistical procedures are able to estimate the necessary values of parameters.

If we do not assume that all the members of one group are similar, we believe that, having left one group, the participant is in an intermediate state for several time steps; and now the participants differ according to this state. Since we are interested in methodological aspects in this regard, we shall consider only two intermediate transitions, although their number can be increased. In our case, dynamic equations will reflect two additional moments of time and the corresponding intermediate groups *Δx, Δy, xx, yy*. In the calculation formulas of the model these variables are recorded with the same index of the moment of time *t*, although by their content they belong to the previous ones: *xx, yy* are one step behind, and *Δx, Δy* are two steps behind.

Those who have fallen (with the probabilities *q1, q2*) under the influence of information channels form the first “intermediate group” (*Δx* and *Δz*); then, having passed through another stage, they get into the next intermediate group (*xx* and *zz*). And only at the next stage they become equal members of an active group. Those who leave the passive groups (*Δy* and *yy*) go through the same procedure. The equations to change the attitude contain logical conditions providing the values of the attitude between zero and *E1* or *E2*.

Then the dynamic equations are as follows:

Model II (basic)

1. $x_{t+1} = x_t + h_0 \cdot (-x_t \cdot \rho + xx_t)$
2. $xx_{t+1} = \Delta x_t$
3. $\Delta x_{t+1} = q1 \cdot \left(r \cdot \frac{x_t}{x_t + z_t} + G1 \cdot cnorm(e1_t) \right) \cdot y_t$
4. $y_{t+1} = h_1 \cdot \left[-q1 \cdot \left(r \cdot \frac{x_t}{x_t + z_t} + G1 \cdot cnorm(e1_t) \right) \cdot y_t - q2 \cdot \left(p \cdot \frac{z_t}{x_t + z_t} + G2 \cdot cnorm(e2_t) \right) \cdot y_t \right] + yy_t$
5. $yy_{t+1} = \Delta y_t$
6. $\Delta y_{t+1} = x_t \cdot \rho + \pi \cdot z_t)$
7. $z_{t+1} = z_t + h_2 \cdot (-\pi \cdot z_t + zz_t)$
8. $zz_{t+1} = \Delta z_t$
9. $\Delta z_{t+1} = q2 \cdot \left(p \cdot \frac{z_t}{x_t + z_t} + G2 \cdot cnorm(e2_t) \right) \cdot y_t$
10. $e1_{t+1} = if\left[e1_t \le 0, 0, if\left[e1_t \ge E1, E1, e1_t + h_3 \left[A1 \cdot (E1 - e1_t) + B1 \cdot y_t \cdot \frac{x_t - z_t}{x_t + z_t} \right] \right] \right]$
11. $e2_{t+1} = if\left[e2_t \le 0, 0, if\left[e2_t \ge E2, E2, e2_t + h_4 \left[A2 \cdot (E2 - e2_t) + B2 \cdot y_t \cdot \frac{z_t - x_t}{x_t + z_t} \right] \right] \right]$

We implemented and calculated the model in the MATHCAD package with the use of conditional data.

Let us consider the dynamics of trajectories with initial conditions $x_0 = z_0 = 100$, $y_0 = 800$, $e1_0 = 1$, $e2_0 = 3$ and at the following values of parameters:

A1=0.28, *A2*=0.28, *B1*=0.05, *B2*=0.05, *E1*=6, *E2*=6, *G1*=0.43, *G2*=0.43, *r*=0.01, p=0.031, *q1*=0.32, *q2*=0.20, *ρ=0.015, π=0.015.*

The results of the calculations are presented in *Table 1* and in *Figure 3.*

The initial quantities of active groups are similar. The initial attitude ($e2_0$=3) in those who contact with the *z*-media is greater than in those who contact with the *x*-media ($e1_0$=1); the importance of social contacts for transitions of the *z*-group is also greater than that of the *x*-group (*p*>*r*). However, due to the greater prevalence of the *x*-media (*q1*>*q2*) the limit attitude and size of the *x*-group exceed the values of their "competitors".

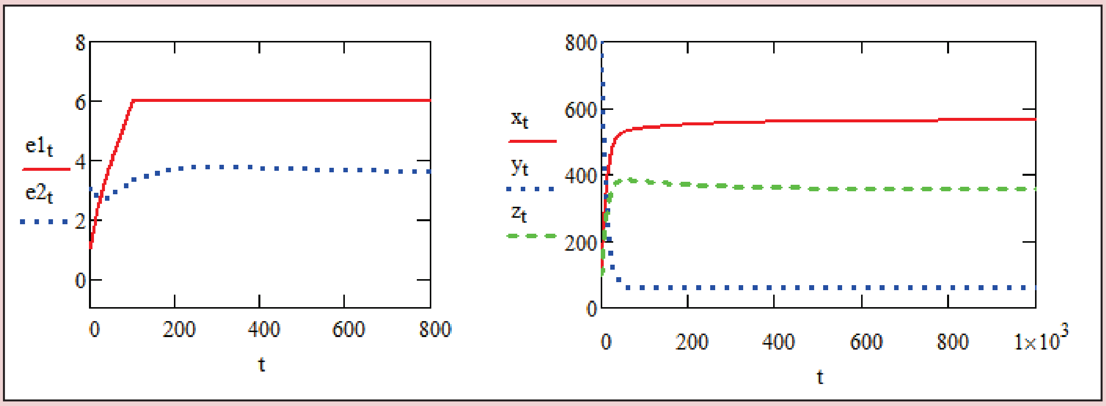
We also see that the number of those who move from one group to another in one time step of the iteration ("intermediate groups") is much less than the number of basic ones, which is natural, since the groups transform

Table 1. Stationary values of the variables

X	*xx*	*Δx*	*y*	*yy*	*Δy*	*z*	*zz*	*Δz*	*e1*	*e2*
563.03	8.445	8.445	60.515	13.794	13.794	356.592	5.35	5.35	6	3.6

Compiled on the basis of our own calculations.

Figure 3. Trajectories of the model variables



Compiled on the basis of our own calculations.

gradually. And the shares of those who move to the active groups are defined by the prevalence of propaganda (*q1, q2*), the probability of contacts (*p ,r*) and the force of impact of the attitude (*G1, G2*). In this virtual world the *x* ideology defeats the alternative ideology pretty quickly.

7. The model with the constant percentage of "unyielding" participants (*Model III*)

To emphasize a certain versatility of our approach, we calculated the trajectories of the basic model under a *nonlinear law of influencing the attitude* on the part of the media according to (1). In addition, we introduced a new parameter δ that denotes the constant share of those members of the active first group ("unyielding") that do not leave it under any value of *t*. Now equations 1 and 6 in *Model II* are replaced by the following ones:

$$x_{t+1} = x_t + h_0 \cdot [(-(1-\delta)x_t \cdot \rho + xx_t)]$$
$$\Delta y_{t+1} = (1-\delta)x_t \cdot \rho + \pi \cdot z_t$$

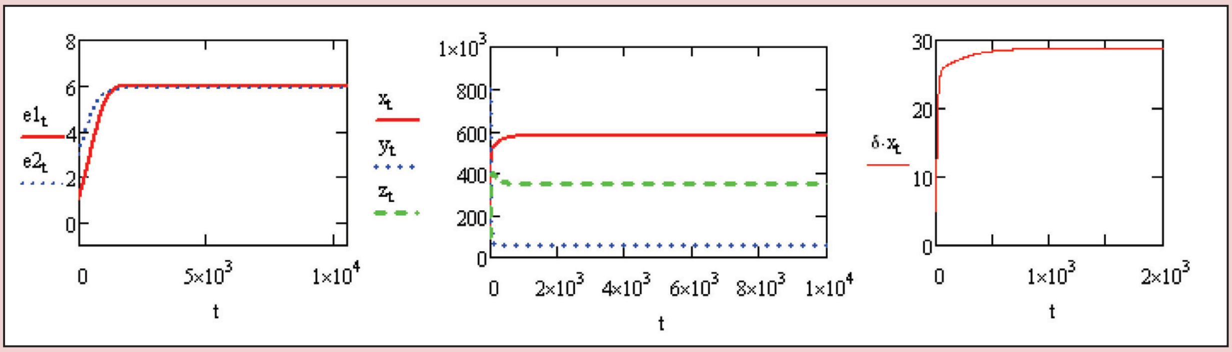
The graphs in *Figure 4* show that the nature of the trajectories of quantities remains virtually unchanged, but both attitudes tend to their limit. The third graph shows that the number of the most "unyielding" (or "conservative") participants quickly stabilizes and does not change.

Let us consider another simulation of a real process at different variants of the values of individual parameters. First, let us assume that the parameters for all groups are the same (*G1=G2, p=r, q1=q2,* $\rho=\pi$), and the initial values of the variables are presented in Column 1 of *Table 2.*

The behavior of the variables *x* and *z* at $t > 0$ does not vary when reaching its final state (see the first line of Table 2) and the neutral group also goes into its stationary state. However, even a slight predominance of one of the initial quantities of the active groups *x*=80, *z*=20 saves the predominance for the whole period (second line), as in the previous case. Significantly, due to the imitation effect, the numerical superiority of the second group exceeded even the initial level of the attitude of the *z*-group ($e1_0=e2_0=2$). This fact shows the role of social contacts when the sizes of groups differ.

Computer-assisted calculations show how we can compensate for the small initial size of one of the groups to ensure its further predominance. First, we can reduce the gap,

Figure 4. Trajectories of variables of Model III



Compiled on the basis of our own calculations.

Table 2. Dependence of stationary values on initial values

Initial	Solution				
x_0; y_0; z_0	x	y	z	e1	e2
50; 900; 50	457.789	78.93	457.789	6	6
80; 900; 20	594.97	102.19	297.485	6	0
Compiled on the basis of our own calculations.					

to level the sizes or even increase the size by increasing the prevalence of the mass media (coefficients *q2*); and second, we can increase the impact of the attitude toward the transition from the neutral group to the active group (*p, G2*). When the prevalence of the media of the second group (*q2*) decreases, then the size of the *x*-group increases at the expense of other groups. When the parameters *p* and *G2* increase, the sizes of the neutral group *y* and the group *z* reduce, while the size of the *x*-group increases. At the simultaneous reduction in the prevalence of the media channels *q1, q2* the sizes of the active groups (*z, x*) reduce, and the size of the neutral group (*y*) increases. Our calculations have confirmed these possibilities.

8. A model with external effects *(Model IV)*

Model III describes the interaction of social groups with information channels, when nothing except these factors influences the opinions and behavior of the group. And under certain parameter values the model is a method of forecasting the observed processes. People's behavior is always influenced by the context they live in: the standard of living, rise in prices, political events in society, etc. These conditions define the fixed values of the model parameters. This may be reflected in a *dependence* of parameters determining probability of behavior on the external environment; such parameters are as follows: to leave the group of activists (ρ, π), to move from the neutral group to an active group (*r, G*), to perceive information with more or less trust (*A, B*). We can also reflect the increase or decrease in the effectiveness of information influence at the expense of additional funds on advertising, propaganda and agitation. Thus, the model becomes a model of control, because knowing how to adjust, for example, the parameters *q* or *G*, we increase the size of the group.

Specifically, in a certain variant calculations under *Model IV* we assume that over time the transitions from the neutral group to the group of *z*-activists experience external influence, which is manifested in the fact that the coefficient *G2* (the effect of the attitude on transitions) periodically increases by some value. As a result, there is a natural increase in the size of the *z*-group. If the change is manifested regularly in the form of a recurring "add-on" *Ψ*, then there is no certain stationary state, and there will be a certain attractor that sets a multitude of possible states. This leads to the oscillatory intensification of the *z*-propaganda and – ceteris paribus – to the predominance of the size of the *z*-group *(Tab. 3, Fig. 5 and Fig. 6)*.

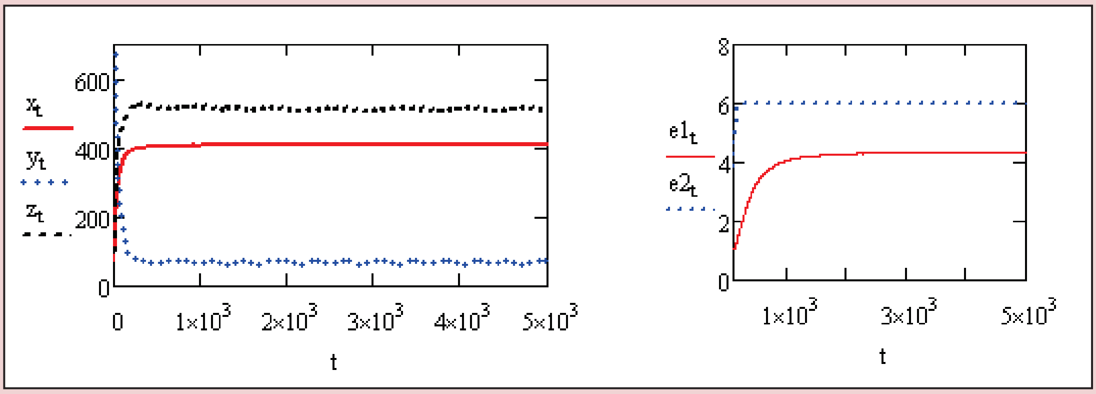
In *Model IV* the value *Ψ* represents an increase in the previous value of the parameter *G;* the change is simulated by a sinusoid in time:

$$\Psi = G2 + d\cdot\Phi(2\cdot\sin(k\cdot t)-1)\cdot 2\cdot\sin(k\cdot t)-1.$$

Table 3. Quasi-stationary values of the variables for *t*=1000

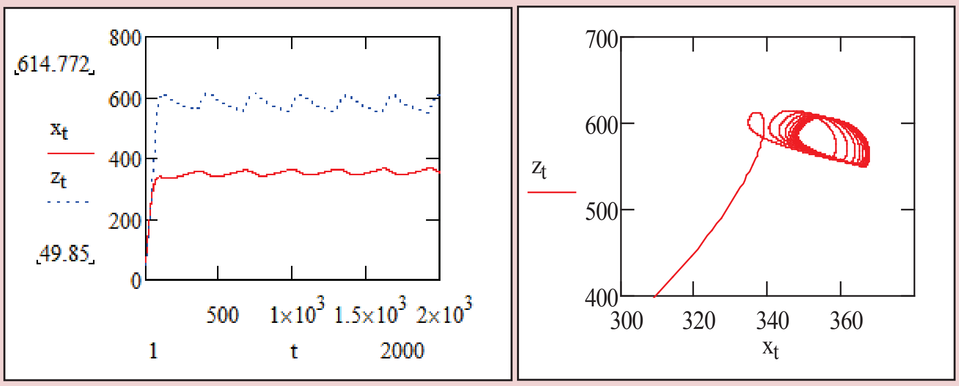
Ψ	*x*	*xx*	*Δx*	*y*	*yy*	*Δy*	*z*	*zz*	*Δz*	*e1*	*e2*
0.757	411.94	6.53	6.50	74.47	13.78	13.78	507.37	10.5	10.98	4.33	6
Compiled on the basis of our own calculations.											

Figure 5. Trajectories of the variables of Model IV



Compiled on the basis of our own calculations.

Figure 6. Dependence of trajectories from periodic external influences



Compiled on the basis of our own calculations.

The coefficient *d* is a multiplier that specifies the value of an additional summand. The function $\Phi(\cdot)$ retains only positive values of the argument. Every $x = \frac{2 \cdot \pi}{k}$ time steps, the value of "infusion" is repeated and the coefficient *k* characterizes this frequency. Naturally, the new values differ from those obtained previously without the external influence.

This model is obtained from *Model II* by adding the initial line to determine *Ψ*, and by altering lines 5 and 10:

$$y_{t+1} = h_1 \cdot \left[- q2 \cdot \left(p \cdot \frac{z_t}{x_t + z_t} + \Psi_t \cdot cnorm(e2_t) \right) \cdot y_t - \ldots \right] + y_t$$

$$\Delta z_{t+1} = q2 \cdot \left(p \cdot \frac{z_t}{x_t + z_t} + \Psi_t \cdot cnorm(e2_t) \right) \cdot y_t$$

The parameters of the model are the same for both active groups with the exception of the introduced *Ψ*.

h_0=0.1, h_1=0.1, h_2=0.1, h_3=0.015, h_4=0.015,

A1=0.28, *A2*=0.28, *B1*=0.06, *B2*=0.06, *E1*=6, *E2*=6, *G1*=0.43, *G2*=0.43, *r*=0.01, *p*=0.01, *q1*=0.2,

q2=0.2, *d*=0.5, *ρ*=0.015, *π*=0.015.

Initial values of the variables: *x*=*z*=50, *y*=900.

Due to external influence, the size of the *z*-group increases unevenly, leading to oscillatory changes in the sizes of other groups. At the same time, the limit values of attitudes remain unchanged, but different in magnitude.

Technically, the same method can be used to consider the impact of uncertainty in the assessment of each parameter. Setting such a parameter as a random variable with known characteristics, we can obtain the set of possible implementations of trajectories and explore the structural stability of the model, as described in our work [2, p. 107-118].

9. Conclusion

The main goal that we tries to achieve in the course of our research was to develop a computer-assisted approach based on a simple mathematical model, the approach that could reflect complex interaction of social groups between themselves and the media shaping an ideological understanding of the world. The urgency of this problem is particularly noticeable when it is clear that social conflict is always preceded by ideological split.

The article proposes an original formalization of the dynamics of the ideological structure of society, when:

1) changing the sizes of the groups is influenced by social attitudes, and the attitudes change depending on the sizes of the groups and the influence of the media.

2) in order to bring Markov-type models closer to reality we introduce "intermediate variables" related to several previous moments of time. This allows us to take into account time-related differences between members of similar groups.

3) forecast and management abilities of the models are increased with the help of external influence factors that we have considered and due to the possibility of adding random perturbations to the model parameters.

It is important to note that the article considers mass processes, which can be described by statistical indicators. Analyzing the forms of organization of group members, which often proves decisive is beyond the scope of our present work. The program we developed allows us, using conditional data about the virtual world, to consider various ways of forming social positions of population, and if real data are available – to design different scenarios of forecast calculations and control actions. In particular, at the model level it is possible to identify the roles and facilitators of the prevalence and availability of media channels and their effectiveness, and social contacts, which in combination leads to various effects. Our experimental calculations demonstrate how the media and prepared social contacts can be used to influence the size of social groups of a particular political orientation. Finally, the proposed procedures can become an underpinning for an "agent-based" model that would simulate ideological confrontation in society and in which the logic of reflection of reality is actually consistent with the logic of our models.

Naturally, the completeness and adequacy of available information is an "Archimedean point" in finding a solution to forecasting and controlling the flows of public consciousness. In turn, this can be implemented only with the help of an active sociological monitoring, rather than separate statistical samples; the monitoring should be followed by competent econometric evaluation of the model parameters. In our opinion, the main part of mathematical correlations in the models we propose, in the presence of the realized (and observed) data, can serve as the basis for econometric equations, which can be used to obtain estimates of the model parameters. And the most difficult thing in this case is that we need to have the data for several subsequent points in time. It is within the scope of the monitoring service.

In conclusion, we would like to note that although our approach focuses on describing the political and ideological confrontation in society, it is applicable to the simulation of situations in different spheres of life, for example, when studying people's attitudes toward the authorities, ideologies of state-minded persons and liberals, strict moralists and advocates of free love, believers and atheists. We plan to extend our approach to the cases with more than two competing groups, consider the social characteristics of groups [24], and the influence of several external factors, and to carry out calculations on the basis of real statistical data. We also plan to develop econometric approaches to assessing the parameters of computer models.

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Information about the Authors

Yurii N. Gavrilets – Doctor of Sciences (Economics), Professor, Head of Laboratory, Central Economics and Mathematics Institute RAS (47, Nakhimovsky Avenue, Moscow, 117418, Russian Federation; e-mail: yurkag@mail.ru)

Iraida V. Tarakanova – Researcher, Central Economics and Mathematics Institute RAS (47, Nakhimovsky Avenue, Moscow, 117418, Russian Federation; e-mail: Itar4@mail.ru)

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