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Neural Network Forecasting Algorithm as a Tool for Assessing Human Capital Trends of the Socio-Economic System

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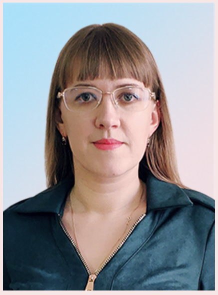

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Abstract. The article addresses the issue of neural network forecasting of human capital size, structure, and dynamics. The object of the research is the socio-economic system. The subject of the study is the practice of applying neural network models to forecasting socio-economic indicators, human capital in particular. The purpose of this work is to apply neural network modeling and adapt its algorithms to build a forecast of the studied indicator for the future. The statistical base is data on demography, the investment volume in human capital of the regional economic system, as well as the indicators of socio-economic development. The investment volume in human capital is determined by budget and private citizens' expenditures. To forecast the human capital dynamics the authors used the values of investment volumes the forecast of which, in turn, is built using a neural network modeling. The neural network model used in this study is a multi-layer fully connected perceptron with a sigmoid logistic activation function.

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Neural network modeling of forecast values of investment volumes has shown its effectiveness. Human capital assessment for the period of 2000–2018 and its forecast for the period of 2019–2023 are based on the example of the regional economic system of the Udmurt Republic. Our calculations show that the highest growth rate of the studied indicator has been demonstrated since 2013, and its further increase is predicted. The results obtained correlate qualitatively with the dynamics of changes in the Russian human development index, determined by the UN experts. The proposed method of calculating and forecasting human capital can be used to assess and compare the socio-economic situation of Russia's regions.

Key words: human capital, neural network modeling, algorithm forecast, investments, socio-economic system.

Introduction

Building development strategy for the socio-economic system which is based on a stable growth of its indicators is an urgent task. When solving this problem, a mathematical modeling tool should be used, as it allows building economic dependencies for the future with a given accuracy.

The strategy for the socio-economic system development involves the construction of schemes for financing industrial and social spheres of activity. The production sphere is characterized by the structure and size of production capital (PC), the methodology for calculating which is well known. In recent years, human capital (HC) has been increasingly used to assess the social sphere which is a leading factor in determining high development rates in modern economy. Consequently, issues related to the human capital study become important and relevant.

The HC concept appeared in science in the 20th century in the works of American scientists-economists J. Mincer [1], T. Schultz [2] and G. Becker [3]. The impetus for the HC theory creation was statistical data on the growth of the developed countries' economies. The analysis of real processes of development and growth in modern conditions has led to the HC study as the main productive and social development factor. For creating the foundations of the human capital theory, the Nobel prizes in Economics were awarded (T. Schultz in 1979 and G. Becker in 1992). S. Kuznets made a significant contribution to the HC theory creation. He showed that human capital is the main dominant factor in the possible stable growth of developing countries' economies [4].

Among the scientists who have made the greatest contribution to the HC theory development are M. Blaug, M. Grossman, M. Perlman, L. Thurow, F. Welch, B. Chizwick, J. Kendrick, R. Solow, R. Lucas, C. Grilliches, I. Fischer, E. Denison and other economists, sociologists and historians. Among modern Russian researchers dealing with the HC problems, we can mention S.A. Dyatlov, R.I. Kapelyushnikov, M.M. Kritskii, S.A. Kurganskii, O.I. Ivanov, and others. There are several approaches to assessing human capital. The models of I. Ben-Porat [5], D. Heckman [6], A.S. Akopyan, V.V. Bushuev and V.S. Golubev [7], S.Yu. Malkov, K.A. Bolotova, and O.I. Davydova [8].

Building a general methodology for the HC calculating is a rather difficult task, as there is a large proportion of subjective assessments when modeling it. There are also different positions on the basis of which the HC concept is formulated, in particular, it can be studied from the point of view of the quality of human life

[9], its ability to perform high-performance activities [10], as the amount of income that a person can receive [11] or as the amount of investment in the social sphere [12]. In 1990, the United Nations Development Program proposed a methodology for assessing the HC level based on the calculation of the human development index[1].

Whatever approach the researchers would chose to assess HC, it is important that at present this indicator should be considered as one of the most important factors that ensure the socio-economic system development [13–17].

In our work, we used an integral economic and mathematical HC which includes its quantitative and qualitative characteristics [18]. Based on this pattern, the HC value is calculated. We performed the HC forecast using a neural network algorithm.

Neural network modeling is one of the mathematical methods for studying various processes and phenomena. It is used for solving problems of data mining and forecasting [19–22].

Initially, artificial neural networks (ANN) were constructed as a result of studying the nervous system of a living organism [23]. Neural network is in a certain way an analogous to the brain in terms of its qualitative structure and the number of neurons it contains.

In recent decades, neural networks have been widely used for forecasting. Forecasting is one of the most popular, but also the most complex tasks of ANN. The forecast error always depends on the selected pattern, the availability and quality of the source data.

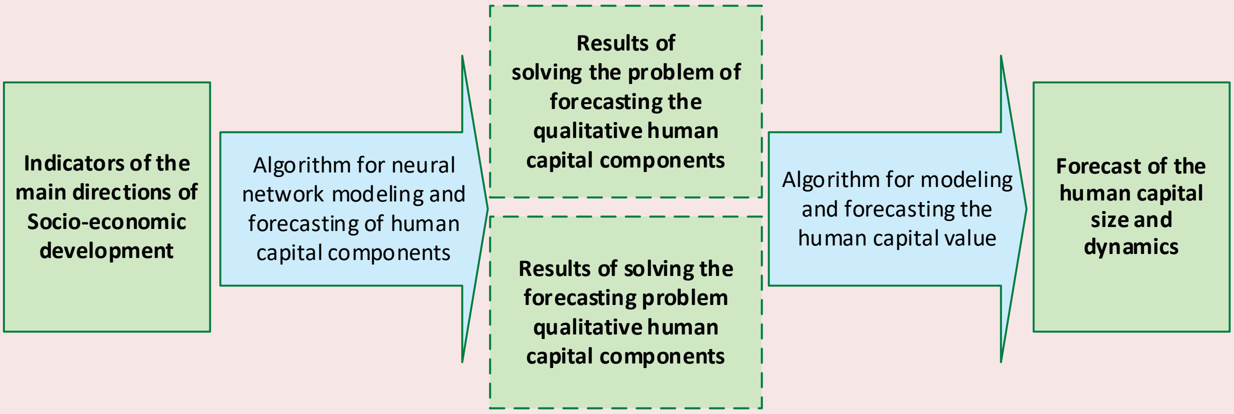
Neural networks have the following advantages in making forecasts: effectiveness in solving unformalized or poorly formalized tasks; resistance to frequent changes in the environment; efficiency when working with a large amount of contradictory or incomplete information. These advantages are relevant including the cases when applying neural networks to the forecasting socio-economic processes and phenomena.

The advantages associated with the use of neural network patterns and their modifications in the analysis of socio-economic processes and phenomena are presented in [24, 25]. The authors show that neural network patterns have the property to take into account the influence of implicit factors and include non-obvious mathematical connections in the study that are difficult to find using classical econometric patterns, the regression ones in particular.

ANN were first mentioned in the 1940s. It is recognized that the neural networks theory was designated as a scientific direction in 1943 in the work of W. McCulloch and W. Pitts [26], where the authors showed that any arithmetic or logical function can be implemented using a simple neural network. Among the fundamental works, we should also highlight the pattern of D. Hebb [27] who proposed the first learning algorithm in 1949. In 1958, F. Rosenblatt constructed a pattern of a perceptron containing a single layer [28]. After some cooling to the ANN, a new wave of interest emerged in the 1970s. Thanks to the research of T. Kohonen, S. Grossberg and D. Anderson, the foundation was completed, on the basis of which it became possible to build and use multi-layer neural networks [29; 30]. In 1974, P. Werbos developed a basic algorithm for training multi-layer neural networks which was widely used in practice [31]. Among the researchers in the ANN, one can also distinguish M. Minsky [32], K. Fukushima [33], J. Hopfield [34], S. Haykin [35], R. Hecht-Nielsen [36], and others. Recent research on the formation of

[1] *The Yearbook of the United Nations*. Available at: https://unyearbook.un.org/

Figure 1. Research structure



Source: own calculations.

neural network modeling algorithms, dynamic series analysis, and forecasting is presented, for example, in [37–40].

We consider the problem of forecasting the human capital of a socio-economic system based on a neural network algorithm. The study structure is shown in *Figure 1*. Indicators of socio-economic development directions are determined in accordance with the work [41]. The neural network modeling algorithm and forecasting of the HC components allows making their forecast. The algorithm for modeling and predicting the HC value uses the results of forecasting the HC components to calculate its total value.

Economic and mathematical pattern of human capital

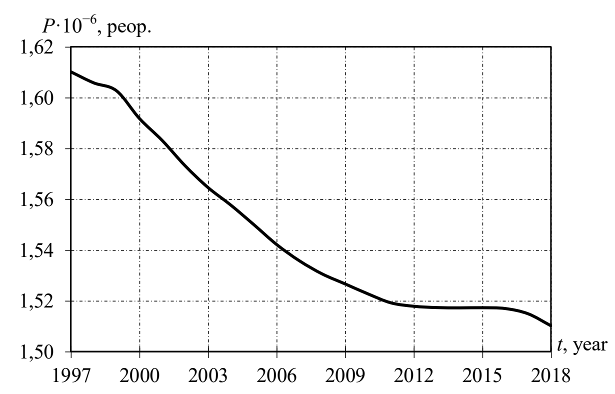
The HC carriers are demographic elements each of which is characterized by age τ at a time t. The number of demographic elements determines the quantitative HC characteristics. To implement an effective demographic policy, it is of great importance to conduct research such as the analysis of modern demographic processes and forecasting the population size and structure based on it, taking into account the specifics of birth rate, life expectancy, migration processes, and mortality.

Figure 2 shows the dynamics of the total population for the 1997–2018 period using the example of one of the regions of the Russian Federation – the Udmurt Republic (UR). *Figure 3* shows the dynamics of fertility and mortality.

The calculation of the HC value should be carried out taking into account the demographic structure. The distribution of demographic elements by age $\rho(t,\tau)$ is important in this case. Under the demographic element here and further we will understand a representative of the population – an individual who at the time t under consideration is characterized by the age τ. *Figure 4* shows the age distribution of the UR population for 2008 and 2018, respectively. The function $\varepsilon(t,\tau)$ sets the percentage of the age τ population participating in social production per t year. The problem of modeling and forecasting demographic dynamics is presented in detail in this work [42].

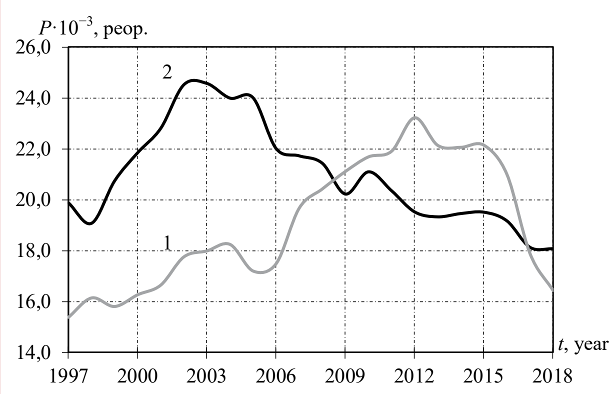
To calculate the human capital value of the economic system *H(t)*, the pattern [18] is used according to which the total value of the

Figure 2. UR population dynamics



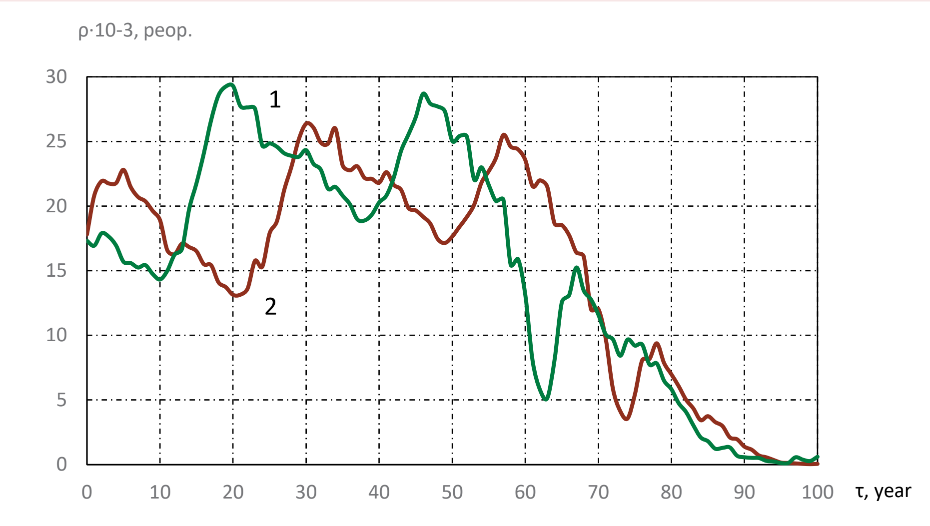
Source: Total population of the Udmurt Republic. Available at: https://udmstat.gks.ru/folder/51924

Figure 3. UR population dynamics of birth rate (1) and death rate (2)



Source: Population of the Udmurt Republic by gender and age. Available at: https://udmstat.gks.ru/folder/51924

Figure 4. The density of the UR population distribution by age: 2008 (1); 2018 (2)



Source: Population movement of the Udmurt Republic. Available at: https://udmstat.gks.ru/folder/51924

population HC participating in social production is determined from the expression:

$$H(t)=\int_0^{\infty}\sum_{i=1}^{3}\alpha_i h_i(t,\tau)\varepsilon(t,\tau)\,\rho(t,\tau)d\tau. \qquad (1)$$

Functions $h_i(t,\tau)$ are the quality HC components: health $i=1$, education $i=2$, and culture $i=3$. Investment in health raise the general level of health in society which contributes, on the one hand, extending the economically active life of individual demographic item, and, on the other hand, the increase in the number of economically active population elements due to the reduction of mortality. Investment in education contributes to improving the overall skills level in the regional labor market, opening up the greatest reserves for improving the modern economy efficiency. Investments in culture improve the human environment, form moral values, and increase the creative potential of the individual which, of course, affects the region's socio-economic development.

The specific (per demographic unit) HC average value is determined by a linear combination:

$$h(t,\tau) = \alpha_1 h_1(t,\tau) + \alpha_2 h_2(t,\tau) + \alpha_3 h_3(t,\tau), \quad (2)$$

$$\alpha_i \in (0,1); \ \sum_{i=1}^{3} \alpha_i = 1, \quad (3)$$

where: α_i are the corresponding weight coefficients for the HC components; values $h_i(t,\tau)$ are measured in monetary units.

During the research, the hypothesis of the equivalence of the HC components was accepted: $\alpha_i = 1/3, i = \overline{1,3}$.

The evolution of each of the HC components is described by the equation:

$$\frac{\partial h_i(t,\tau)}{\partial t} + \frac{\partial h_i(t,\tau)}{\partial \tau} = -v_i h_i(t,\tau) + s_i(t,\tau) + p_i(t,\tau). \quad (4)$$

In formula (4), the following notations are used for each HC component: $s_i = s_i(t,\tau)$ – specific state expenditures; $p_i = p_i(t,\tau)$ – specific private investment; $v_i = v_i(t,\tau)$ – the coefficient of disposal which is estimated using the identification algorithm [43].

The initial conditions for $t = t_0$ are as follows:

$$h_i(t_0,\tau) = h_{i0}(\tau), \ (i = 1, 2, 3), \quad (5)$$

where: $h_{i0}(t)$ – known functions.

At the left end of the demographic curve, the boundary conditions are as follows:

$$h_i(t,0) = 0, \ (i = 1, 2, 3); \quad (6)$$

at the right end, where $i = 1, 2$ the following should be written:

$$h_i(t,\infty) \approx h_i(t,\tau_m) = 0, \quad (7)$$

where: $\tau_m = \tau_m(t)$ – survival time δ of the population's percent (δ = 1–5%).

It is obvious that the coefficients of disposal v_i are weakly time-dependent. The age dependence for functions $v_i = v_i(\tau)$ $(i = 1, 2)$ is taken as follows:

$$\nu_i(\tau) = \begin{cases} 0, & \tau \le \tau_{ai}, \\ (s_i(t,\tau) + p_i(t,\tau))\{\exp[\varepsilon_i(\tau - \tau_{ai})] - 1\}, & \tau_{ai} \le \tau \le \tau_m, \end{cases} \quad (8)$$

Where the unknown parameters (s_i, ε_i) are determined from the conditions:

$$(s_i(t,\tau) + p_i(t,\tau))\{\exp[\varepsilon_i(\tau_m - \tau_{ai})] - 1\} = 1, \quad (9)$$

$$\int_0^{\tau_m} [s_i(t,\tau) + p_i(t,\tau)] d\tau = \int_{\tau_{ai}}^{\tau_m} \{(s_i(t,\tau) + p_i(t,\tau))(\exp[\varepsilon_i(\tau - \tau_{ai})] - 1)\} h_i(t,\tau) d\tau. \quad (10)$$

Here τ_{ai} is the upper limit of the active period of physical condition (i=1) or work activity (i=2).

When performing specific calculations for the UR economic system, we have analyzed demographic data for the period of 2000–2018, and found that the age limit of the economically active age of the UR population is 20–60 years old.

Unlike other components, the cultural HC component is not subject to disposal, so $v_3 \equiv 0$.

For the equation (4) that sets the evolution of the HC components, the age distribution of the specific components of state budget expenditures $s_i(t,\tau)$ is determined by the formulas:

$$s_i(t,\tau) = \sum_{Ni} \frac{S_{Ni}(t)}{\int_{\tau_{1Ni}}^{\tau_{2Ni}} \rho(t,\tau) d\tau},$$

$$S_{Ni}(t) = \begin{cases} S_{Ni}(t,\tau), & \tau \in [\tau_{1Ni}, \tau_{2Ni}], \\ 0, & \tau \notin [\tau_{1Ni}, \tau_{2Ni}]. \end{cases} \quad (11)$$

Here $S_{Ni}(t)$ is the amounts budgeted for the corresponding item of expenditure N_i (N_i – numbering of budget items spent on health (i=1), education (i=2), and the development of the cultural HC component (i=3)). In accordance with formula (11), these amounts will be distributed evenly over the corresponding periods of a person's life [τ_{1Ni}, τ_{2Ni}] and the number of demographic units in these periods.

For equation (4), the age distribution of the specific components of private expenditures $p_i(t, \tau)$ aimed at HC increasing is written by analogy with (11), assuming that a person uses various amounts of money spent on health (i=1), education (i=2), and culture in different periods of life (i=3):

$$p_i(t,\tau)=\sum_i \frac{P_i(t)}{\int_{\tau_{1i}}^{\tau_{2i}} \rho(t,\tau)d\tau}, \quad P_i(t)=\begin{cases} P_i(t,\tau), & \tau\in[\tau_{1i},\tau_{2i}],\\ 0, & \tau\notin[\tau_{1i},\tau_{2i}]. \end{cases} \qquad (12)$$

Algorithm for neural network forecasting of human capital

Let us build a neural network for solving the problem of predicting human capital. *Figure 5* shows a neural network element – a neuron with m inputs and an output. This construction of a neuron is called a perceptron. An artificial neural network is a way to assemble neurons into a network to solve certain tasks. The trained artificial neuron works as follows: each input is multiplied by certain weights; then everything is summed and run through a nonlinear activation function, and the result is fed to the output. All the neurons are collected into layers. Basic computing operations are performed on internal hidden layers.

Another task is to train the neuron. Training consists in finding the correct weight information which is adjusted based on the task and error (the difference between the calculated and ideal output values). For this purpose, the error back propagation algorithm is used.

The input data is summed with weights w_j: $D=\sum_{j=1}^{m} w_j x_j$. The neuron's output has a logistic activation function $f(D)$ (Fig. 5) that converts the weighted sum D of the incoming signal (*Fig. 6*).

To forecast the investment volume in the social sphere, we construct a complex k-layer neural network with m inputs and l outputs (*Fig. 7*). The input data in the neural network pattern are the volumes of budget $\{S_i\}_{i=1}^{n=3}$ and private $\{P_i\}_{i=1}^{n=3}$ investment in the HC, the indicators of socio-economic development directions $\{I_i\}_{i=1}^{n=8}$. The deflator index K is used to account for inflationary processes. The output data of the neural network is the projected monthly volumes of budget and private investments in HC: $\{\tilde{S}_i\}_{i=1}^{n=3}$ and $\{\tilde{P}_i\}_{i=1}^{n=3}$.

For our task, we will consider budget investments in healthcare (x_1), education (x_2), and culture (x_3), as well as private investments in healthcare (x_4), education (x_5), and culture (x_6). The amount of investment in the social sphere

Figure 5. Artificial neuron

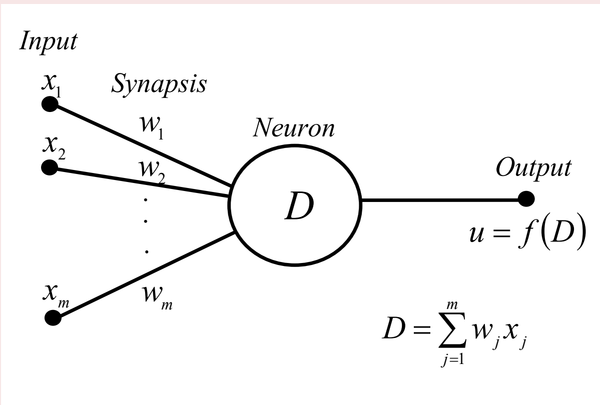


Figure 6. Activation function

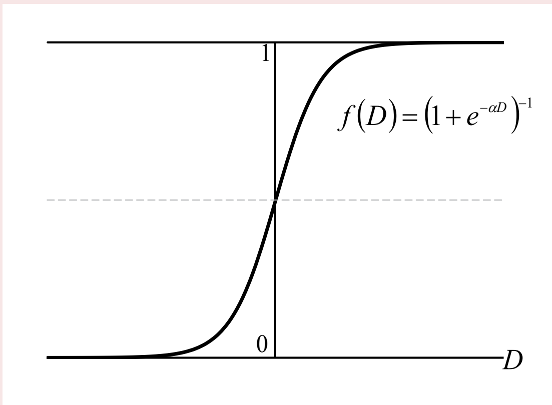
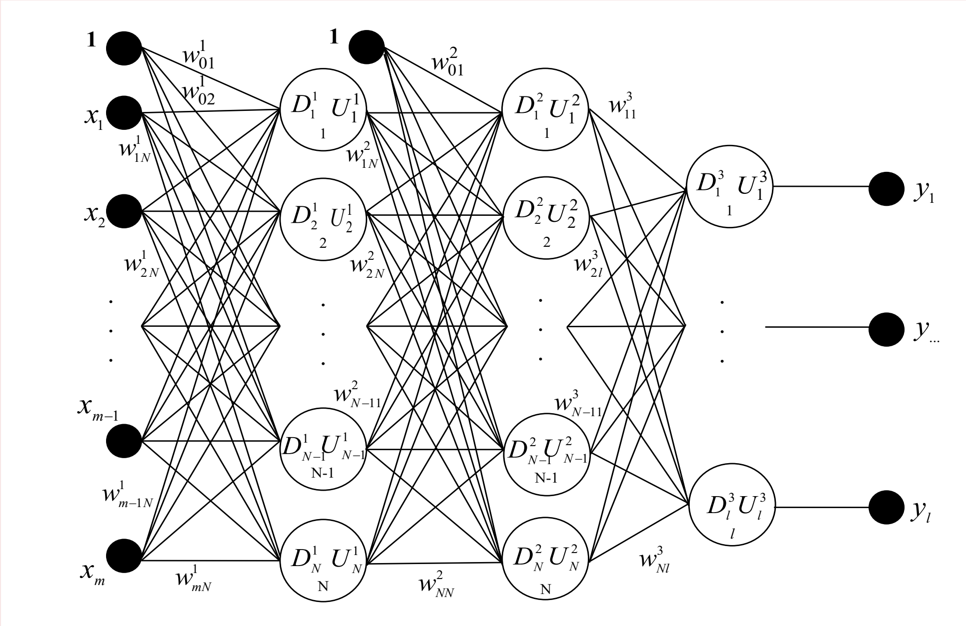


Figure 7. Neural network pattern used to forecast investments in human capital



Source: own calculations.

depends on many factors and environmental conditions. In the work of S.A. Ayvazyan [41] identified indicators that most strongly affect the quality of the social sphere, namely: gross regional product (x_7), average per capita monetary income (x_8), production of goods and services per capita (x_9), the area of housing commissioned (x_{10}), the number of registered crimes (x_{11}), natural population growth (x_{12}), mortality in working age (x_{13}), the minimum required monthly income (x_{14}).

Thus, $\{S_i\}_{i=1}^{n=3} = \{x_1, x_2, x_3\}$, $\{P_i\}_{i=1}^{n=3} = \{x_4, x_5, x_6\}$, $\{I_i\}_{i=1}^{n=8} = \{x_7, x_8, x_9, x_{10}, x_{11}, x_{12}, x_{13}, x_{14}\}$, $K=x_{15}$.

We will construct a neural network prediction algorithm for any case with any number of input variables. Figure 7 shows the neural network pattern underlying the algorithm.

Each layer of the neural network pattern contains N_p neurons $p=1,\ldots,k$.

We will use the following notation: w_{ij}^p the weight coefficient of communication connecting the signal coming out of the (p-1) layer of the i-th neuron and entering the j-th neuron of the p-th layer. For each layer, the coefficients are represented as a matrix with the size $(N_{p-1}+1)\times N_p$:

$$\vec{W} = \left(w_{ij}^p\right),\ p=1,..,k;\ i=0,..,N_{p-1};\ j=1,..,N_p. \quad (13)$$

From an algorithmic point of view, the output values of the zero layer u_j^0 should be equated to the signals entering the neural network x_j, $x_0 \equiv 1$:

$$u_j^0 = x_j,\ j=0,..,m. \quad (14)$$

In other layers, the output values of neurons are calculated:

$$u_0^p = 1,\ u_j^p = f\left(d_j^p\right)\ p=1,..,k,\ j=1,..,N_p, \quad (15)$$

where $f\left(d_j^p\right)$ – nonlinear activation function of the, $f(t)=\left(1+e^{-\alpha t}\right)^{-1}$, α – a coefficient.

Let us denote d_j^p as the input signal in j-th neuron of the p-th neuron layer which is determined by the weighted sum of the incoming signals:

$$d_j^p = \sum_{i=0}^{N_{p-1}} w_{ij}^p u_i^{p-1},\ j=1,.,N_p. \quad (16)$$

The output values of the last k-th layer should match y_j:

$$y_j = u_j^k ,\ j = 1,..,l , \quad (17)$$

The training process consists of adjusting the weight coefficients w_{ij}^p. Based on the information about the values of variables at known points in time, the network determines their most likely values for the future. Statistical information on socio-economic indicators is divided into two sets: the training set and the test set which is a section of the retro-forecast.

To train the network, input data is fed to the inputs $\boldsymbol{x}_q = (x_{q1}, x_{q2},...,x_{qm})$ and the output values of the network are compared with the ideal (actually set) values, $\boldsymbol{r}_q = (r_{q1}, r_{q2},...,r_{ql})$, $q=1,...,n$.

The training set of data is used to implement an algorithm for training a multi-layer neural network using the method of back propagation of errors which is related to gradient optimization methods [44]. To determine the network weight coefficients $\vec{W} = \left(w_{ij}^p\right)$, the network training error is calculated using the formula:

$$E_q\left(\vec{W}\right) = \frac{1}{2}\sum_{j=1}^{l}\left(y_{qj} - r_{qj}\right)^2 ,\ q = 1,..,n, \quad (18)$$

where y_{qj} is j-th output when the input of the q-th image.

When submitting the q-th observation, the coefficients will change as follows:

$$\vec{W}(q) = \vec{W}(q-1) + \left(-\lambda \cdot \nabla E_q\right), \quad (19)$$

where $\vec{W}(q)$ – the vector state $\vec{W}$ after training the network by the q-th observation; $\lambda \in (0;1]$ – training rate of the network; ∇E_q – gradient of function $E_q\left(\vec{W}\right)$, when the input is submitted q-th image:

$$\nabla E_q = \left(\frac{\partial E_q}{\partial w_{ij}^p}\right),\ p = 1,..,k,\ i = 0..N_{p-1},\ j = 1,...,N_p . (20)$$

In the component form, the expression (20) is represented as:

$$w_{ij}^p(q) = w_{ij}^p(q-1) + \Delta w_{ij}^p ,\ \Delta w_{ij}^p = -\lambda \frac{\partial E_q}{\partial w_{ij}^p} . \quad (21)$$

Vector components (21) are written as follows:

$$\frac{\partial E_q}{\partial w_{ij}^p} = \frac{\partial E_q}{\partial u_j^p}\frac{\partial u_j^p}{\partial d_j^p}\frac{\partial d_j^p}{\partial w_{ij}^p}, \quad (22)$$

where partial derivative $\frac{\partial u_j^p}{\partial d_j^p}$ in accordance with the derivative of the logistic function $\frac{\partial f(t)}{\partial d} = \alpha\, f(t)(1 - f(t))$ is represented as:

$$\frac{\partial u_j^p}{\partial d_j^p} = \alpha \cdot u_j^p \cdot \left(1 - u_j^p\right). \quad (23)$$

Let us introduce a new variable δ_j^p as follows:

$$\delta_j^p = \frac{\partial E_q}{\partial u_j^p}\frac{\partial u_j^p}{\partial d_j^p} , \quad (24)$$

$$\frac{\partial E_q}{\partial u_j^p} = \sum_{i=1}^{N_{p+1}} \frac{\partial E_q}{\partial u_i^{p+1}}\frac{\partial u_i^{p+1}}{\partial d_i^{p+1}}\frac{\partial d_i^{p+1}}{\partial u_j^p} = \sum_{i=1}^{N_{p+1}} \frac{\partial E_q}{\partial u_i^{p+1}}\frac{\partial u_i^{p+1}}{\partial d_i^{p+1}} w_{ji}^{p+1}. \quad (25)$$

Then δ_j^p we can recursively calculate through the data of the $(p+1)$-th layer δ_j^{p+1}:

$$\delta_j^p = \left[\sum_{i=1}^{N_{p+1}} \delta_i^{p+1} w_{ji}^{p+1}\right] \cdot \alpha \cdot u_j^p \cdot \left(1 - u_j^p\right). \quad (26)$$

When $p=k$ from (16), (23) and (24), equating $u_j^k = y_{qj}$, we find:

$$\delta_j^k = [u_j^k - r_{qj}] \cdot \alpha \cdot u_j^k \cdot (1 - u_j^k) . \quad (27)$$

The last multiplier in formula (22), according to (16), is equal to: $\frac{\partial d_j^p}{\partial w_{ij}^p} = u_i^{p-1}$.

As a result, based on the formulas (21), (22), (24) we get the difference scheme:

$$w_{ij}^p(q) = w_{ij}^p(q-1) - \lambda \delta_j^p u_i^{p-1} . \quad (28)$$

In order to train the network, you need to normalize the input and output data in the area of their definition. If it is known that $x_j \in [a_j - h_j;\ b_j + h_j]$, then the normalized input data has the following form:

$$\bar{x}_{qj} = \frac{x_{qj} - \left(b_j + a_j\right)/2}{\left(b_j - a_j\right)/2 + h_j},\ q = 1,...,n \cdot \quad (29)$$

Table 1. Budget and private investments aimed at the UR human capital development for 2000–2018 in the current prices

Indicator	Time frame, year	Year, mln. rub.									
		2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Expenditures of the UR consolidated budget and territorial state extra-budgetary funds[1)]											
Budget investments in health and sport		**1459.3**	**1652.8**	**2492.5**	**2704.0**	**3189.9**	**4790.4**	**6792.7**	**13846.9**	**15705.2**	**11632.4**
Health care	$1 \leq \tau \leq \tau_m$	1382.3	1593.7	2418.4	2495.5	2971.6	4537.7	6434.3	13116.4	14876.6	9697.7
Sport and physical education	$3 \leq \tau \leq \tau_m$	42.0	53.7	65.8	81.1	83.8	137.9	195.5	398.6	452.1	240.7
Other expenses	$1 \leq \tau \leq \tau_m$	35.0	5.4	8.3	127.4	134.5	114.8	162.8	331,9	376.4	1694.0
Budget investment in education		**2125.3**	**2535.7**	**3732.6**	**3954.6**	**4781.9**	**5425.5**	**7529.0**	**9224.9**	**10386.9**	**12386.9**
Preschool education	$3 \leq \tau \leq 6$	528.7	670.2	926.9	963.8	1144.8	1349.6	1872.8	2294.7	2583.7	3029.3
General education	$7 \leq \tau \leq 17$	1333.1	1566.3	2394.0	2448.4	3020.2	3403.2	4722.6	5786.4	6515.3	6752.7
Elementary vocational education	$14 \leq \tau \leq 17$	2.7	4.2	5.3	5.5	6.6	6.9	9.6	11.7	13.2	711.0
Intermediate vocational education	$18 \leq \tau \leq 21$	60.4	73.4	107.0	109.6	139.2	154.2	214.0	262.2	295.2	417.1
Retraining and developing skills	$25 \leq \tau \leq 59$	16.6	20.8	28.7	32.5	37.3	42.3	58.6	71.9	80.9	58.1
Higher professional education	$18 \leq \tau \leq 22$	5.3	9.4	8.8	9.4	10.5	13.4	18.6	22.8	25.7	–
Youth policy and health care for children	$3 \leq \tau \leq 59$	178.6	204.9	261.8	385.4	423.2	455.9	632.7	775.2	872.9	207.7
Other issues in education	$3 \leq \tau \leq \tau_m$	–	–	–	–	–	–	–	–	–	1211.0
Budget investment in culture		**307.7**	**319.8**	**475.5**	**584.6**	**661.9**	**2433.0**	**3099.3**	**4204.6**	**3230.0**	**2102.1**
Culture	$3 \leq \tau \leq \tau_m$	235.7	252.9	408.5	435.7	526.7	1874.7	2388.1	3239.7	2488.8	1397.6
Cinema	$3 \leq \tau \leq \tau_m$	16.2	3.2	4.0	3.4	3.5	13.5	17.2	23.3	17.9	0.2
Broadcast media	$3 \leq \tau \leq \tau_m$	28.3	34.9	24.9	79.9	72.1	298.8	380.6	516.3	396.6	78.9
Periodicals and publishing houses	$7 \leq \tau \leq \tau_m$	27.5	28.8	35.1	60.1	50.4	217.7	277.3	376.2	289.0	97.2
Other expenses in culture and mass media	$3 \leq \tau \leq \tau_m$	–	–	–	5.5	9.2	28.4	36.1	49.0	37.6	528.2
Total		**3892.3**	**4508.3**	**6700.6**	**7243.2**	**8633.7**	**12648.9**	**17421**	**27276.4**	**29322.1**	**26121.4**
Expenditures from the Federal budget of the Russian Federation and state extra-budgetary funds in UR[2)]											
Budget investment in health care	$1 \leq \tau \leq \tau_m$	*	*	*	*	*	*	*	*	*	*
Budget investment in education	$1 \leq \tau \leq \tau_m$	*	*	*	*	*	*	*	*	*	*
Budget investment in culture	$1 \leq \tau \leq \tau_m$	*	*	*	*	*	*	*	*	*	*
Total		*	*	*	*	*	*	*	*	*	*
Private investment of the UR population[3)]											
Private investment in health care	$1 \leq \tau \leq \tau_m$	412.2	654.0	875.7	1269.6	1677.3	2355.7	3080.0	3871.1	4722.8	4867.7
Private investment in education	$1 \leq \tau \leq \tau_m$	250.2	305.9	342.5	366.9	482.5	674.4	877.3	1098.0	1335.0	2436.6
Private investment in culture and leisure	$1 \leq \tau \leq \tau_m$	423.2	936.2	1249.7	2479.8	3276.2	4601.2	6015.9	7561.1	9224.6	11565.5
Total		**1085.6**	**1896.1**	**2467.9**	**4116.3**	**5436.0**	**7631.3**	**9973.1**	**12530.1**	**15282.4**	**18869.7**

Continuation of Table 1

Indicator	Time frame, year	Year, mln. rub.								
		2010	2011	2012	2013	2014	2015	2016	2017	2018
Expenditures of the UR consolidated budget and territorial state extra-budgetary funds										
Budget investments in health and sport		**11966.7**	**15018.0**	**21482.0**	**23095.2**	**23801.1**	**26477.7**	**27383.1**	**21195.2**	**24996.2**
Health care	$1 \leq \tau \leq \tau_m$	10500.7	13915.3	20234.3	21418.6	22390.7	25107.9	25418.6	19362.0	23224.9
Sport and physical education	$3 \leq \tau \leq \tau_m$	680.9	1102.7	1247.7	1676.6	1410.4	1369.8	1964.5	1833.2	1741.3
Budget investment in education		**13419.9**	**15300.3**	**19718.0**	**24802.1**	**26549.2**	**26349.0**	**26914.0**	**26469.3**	**31207.0**
Preschool education	$3 \leq \tau \leq 6$	3306.2	3909.7	5201.5	7654.5	8454.7	7900.5	7518.0	7373.2	9596.1
General education	$7 \leq \tau \leq 17$	7152.0	8720.9	10841.2	12837.3	14041.4	14471.7	14920.4	13323.1	14867.6
Elementary vocational education (since 2017 – additional education for children)	$14 \leq \tau \leq 17$	666.6	641.5	703.1	696.5	–	–	–	1673.3	2003.3
Intermediate vocational education	$18 \leq \tau \leq 21$	455.0	564.4	1131.9	1218.7	1874.3	1820.1	1958.5	1908.9	2261.7
Retraining and developing skills	$25 \leq \tau \leq 59$	55.2	51.7	78.5	80.0	83.5	84.9	82.8	82.2	89.9
Higher professional education	$18 \leq \tau \leq 22$	–	–	–	–	–	–	–	–	–
Youth policy	$3 \leq \tau \leq 59$	516.2	537.6	323.3	330.0	338.9	268.0	634.3	588.4	729.8
Other issues in education	$3 \leq \tau \leq \tau_m$	1271.3	867.4	1438.5	1985.2	1756.4	1803.8	1800.0	1520.2	1658.6
Other expenses in health care and sport	$1 \leq \tau \leq \tau_m$	785.1	–	–	–	–	–	–	–	–
Budget investment in culture		**1916.9**	**2211.4**	**2460.3**	**2768.9**	**3679.7**	**3634.4**	**3666.0**	**3868.5**	**4623.0**
Culture	$3 \leq \tau \leq \tau_m$	1642.3	1861.2	2039.2	2230.4	3084.7	2965.5	2904.2	3043.8	3444.7
Cinema	$3 \leq \tau \leq \tau_m$	107.9	143.2	201.4	354.7	407.4	443.0	487.1	565.2	905.7
Broadcast media	$3 \leq \tau \leq \tau_m$	83.6	95.6	104.5	83.0	80.5	84.5	105.5	96.8	80.3
Periodicals and publishing houses	$7 \leq \tau \leq \tau_m$	78.9	106.7	111.5	96.0	104.3	120.5	143.,8	145.6	175.9
Other expenses in culture and mass media	$3 \leq \tau \leq \tau_m$	4.2	4.7	3.7	4.8	2.8	20.9	25.4	17.1	16.4
Total		**27303.5**	**32529.7**	**43660.3**	**50666.2**	**54030.0**	**56461.1**	**57963.1**	**51533.0**	**60826.2**
Expenditures from the Federal budget of the Russian Federation and state extra-budgetary funds in UR										
Budget investment in health care and sport	$1 \leq \tau \leq \tau_m$	*	*	*	*	*	987,3	1150,1	960,4	1065,1
Budget investment in education	$1 \leq \tau \leq \tau_m$	*	*	*	*	*	505.3	618.1	888.1	1440.7
Budget investment in culture	$1 \leq \tau \leq \tau_m$	*	*	*	*	*	10.2	13.4	44.4	50.9
Total			*	*	*	*	**1502.8**	**1781.6**	**1892.9**	**2556.7**
Private investment of the UR population										
Private investment in health care and sport	$1 \leq \tau \leq \tau_m$	5996.2	7155.8	7822.7	9041.0	9629.1	9826.9	10726.4	11644.3	11952.0
Private investment in education	$1 \leq \tau \leq \tau_m$	2410.2	2455.7	2912.5	2434.5	2387.0	2680.2	2413.8	2339.7	2553.5
Private investment in culture and leisure	$1 \leq \tau \leq \tau_m$	12525.5	13996.6	16033.8	18089.3	17995.9	19091.4	19560.4	21168.9	24724.8
Total		**20931.9**	**23608.1**	**26769.0**	**29564.7**	**30011.9**	**31598.5**	**32700.7**	**35152.8**	**39230.4**

*no statistics available.

Sources:

[1] Consolidated budgets of the entities of the Russian Federation and budgets of territorial state extra-budgetary funds. Available at: http://www.roskazna.ru/ispolnenie-byudzhetov/konsolidirovannye-byudzhety-subektov/

[2] Execution of federal budget funds in the Udmurt Republic. Available at:http://udmurtia.roskazna.ru/ispolnenie-byudzhetov/federalnyy-byudzhet/godovoy-otchet-ob-ispolnenii-byudzheta/

[3] Household income, expenditure and consumption. Available at: https://www.gks.ru/compendium/document/13271

TTable 2. Dynamics of basic indicators of the main directions of the UR socio-economic development for 2000–2018

Indicator	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Gross regional product (in current prices), mln. rub.[1]	53307.4	65551.4	78346.3	89034.5	100833.1	139995.3	164848.5	205647.4	243135.5	230938.3
Per capita monetary income [2] (in current prices), rub. per month	1478.4	1974.5	2466.7	3098.1	4657.0	6373.0	7798.1	7823.8	9586.6	11060.0
Production of goods and services per capita[3] (in current prices), thsd. rub. per capita	33.5	41.4	49.8	56.9	64.7	90.3	106.9	133.9	158.9	151.3
Housing commissioning [4], thsd. sq. m.	257	277	354	315	361	372	424	483	486	465
Number of registered crises[5], thsd. items	30.1	30.5	24.5	33.0	37.0	54.2	65.3	53.0	44.9	38.8
National population growth[6], people	-5596	-6174	-6774	-6589	-5756	-6816	-4531	-2060	-1015	882
Working-age mortality[6], people	4498	5120	5456	5853	5859	5781	4912	4753	4553	3955
Minimum required monthly income (average per household)[7], thsd. rub. per month	*	*	*	*	*	*	*	*	*	*
Index-deflator[8] (to the previous year)	1.376	1.165	1.156	1.138	1.203	1.193	1.152	1.138	1.180	1.020

Continuation of Table 2

Indicator	2010	2011	2012	2013	2014	2015	2016	2017	2018
Gross regional product (in current prices), mln. rub. [1]	274578.1	335984.0	372782.7	405126.4	450548.9	517999.8	531855.8	556190.5	600000.0
Per capita monetary income[2] (in current prices), rub. per month	12983.6	14452.3	16693.6	18660.3	21197.5	24454.5	23882.8	23925.4	24418.6
Production of goods and services per capita[3] (in current prices), thsd. rub. per capita	180.3	221.1	245.6	267.0	296.9	341.4	350.6	367.1	398.0
Housing commissioning [4], thsd. sq. m.	482.0	504.9	516.1	545.6	612.3	6485	650.1	658.6	717.1
Number of registered crises[5], thsd. items	32.7	28.0	27.4	27.0	26.9	29.5	26.2	24.2	25.3
National population growth[6], people	584	1547	3699	2806	2599	2630	1822	-258	-1656
Working-age mortality[6], people	4063	3761	3455	3211	3176	2949	2878	2584	2412
Minimum required monthly income (average per household)[7], thsd. rub. per month	*	45.018	*	57.704	60.644	65.113	67.518	67.991	*
Index-deflator[8] (to the previous year)	1.142	1.159	1.091	1.054	1.075	1.076	1.032	1.054	1.103

* no statistics available.

Sources:

1) Gross regional product in the Udmurt Republic. Available at: https://udmstat.gks.ru/folder/51922
2) Standard of living in the Udmurt Republic. Available at: https://udmstat.gks.ru/folder/51930
3) Housing commissioning in the Udmurt Republic. Available at: http://www.minstroy.ru/taxonomy/term/480
4) Russia's regions. Main characteristics of the entities of the Russian Federation. Available at: https://www.gks.ru/folder/210/document/13205
5) Population of the Udmurt Republic. Available at: https://udmstat.gks.ru/folder/51924
6) Results of the Federal statistical observations on socio-demographic problems. Available at: https://www.gks.ru/free_doc/new_site/inspection/itog_inspect1.htm
7) Results of the Federal statistical observations on socio-demographic problems. Available at: https://www.gks.ru/free_doc/new_site/inspection/itog_inspect1.htm
8) National accounts. Available at: https://www.gks.ru/accounts

If it is known that the changes of i-th of the output function is within $[\varphi_i^{\min}, \varphi_i^{\max}]$, then normalized output data has the following form:

$$\bar{r}_{qi} = \frac{r_{qi} - \varphi_i^{\min}}{\varphi_i^{\max} - \varphi_i^{\min}}, \; q = 1,...,n \; . \qquad (30)$$

To get the actual values of the output data, it is necessary to do the reverse conversion:

$$y_{qi} = \varphi_i^{min} + \bar{y}_{qi}\left(\varphi_i^{max} - \varphi_i^{min}\right), \; q = 1,...,n, \quad (31)$$

where y_{qi} – real function value; $\bar{y}_{qi}$ – i-th output normalized value of the function when the network input is the q-th image.

The quality of network training is determined by the training error using the formula:

$$\widetilde{E}(\vec{W}) = 100\sqrt{\frac{2}{l \cdot n}\sum_{q=1}^{n} E_q(\vec{W})}. \qquad (32)$$

The calculation error is determined by the formula:

$$\widetilde{\varepsilon} = \frac{1}{M_y}\sum_{y\in[2000,2018]}\frac{\left|y^* - y\right|}{y}, \qquad (33)$$

where M_y – the number of specified indicator points y on the time axis; y^* – values obtained from the pattern; y – real statistic data.

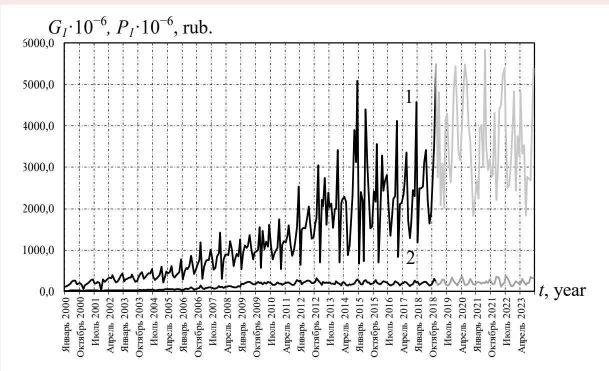
Results of neural network forecasting of human capital

We will perform human capital calculations on the example of one of the regions of the Russian Federation – the socio-economic system of the Udmurt Republic (UR). In the constructed neural network pattern for our example, the number of input neurons N=16 $(1, \{S_i\}_{i=1}^{n=3}, \{P_i\}_{i=1}^{n=3}, \{I_i\}_{i=1}^{n=8}, K)$, the number of hidden layers is equal to two (see *Figure 7*). Statistics on these indicators for UR are presented in *Tables 1, 2.*

Table 1 shows the annual budget and private investments aimed at the UR human capital development for the period 2000–2018 according to the Russian Federal Treasury, and the Territorial Office of the Federal Treasury for the Udmurt Republic and the Federal State Statistics Service of the Russian Federation.

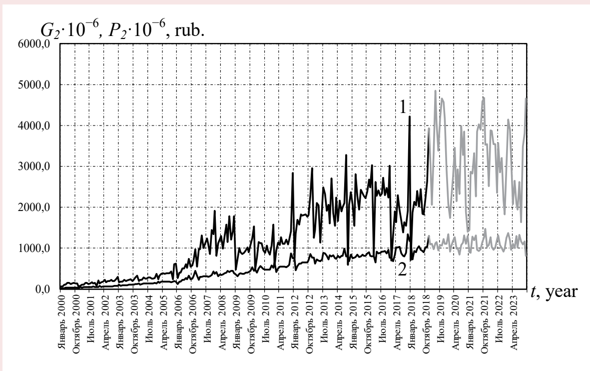
The dynamics of indicators of the UR socio-economic development according to the data of the Federal State Statistics Service, the Territorial State Statistics Service of the UR, as well as the Federal Statistical Observations on socio-demographic problems are shown in *Table 2.*

Figure 8. Investment dynamics in the UR education in 2000–2018 and their forecast in 2019–2023: budget (1), private (2)



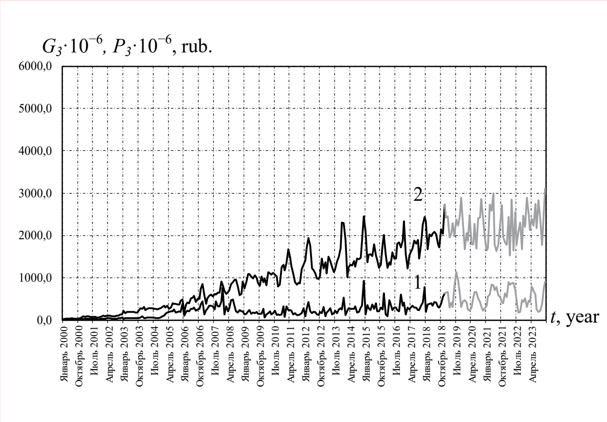
Source: own calculations.

Figure 9. Investment dynamics in the UR healthcare in 2000–2018 and their forecast in 2019–2023: budget (1), private (2)



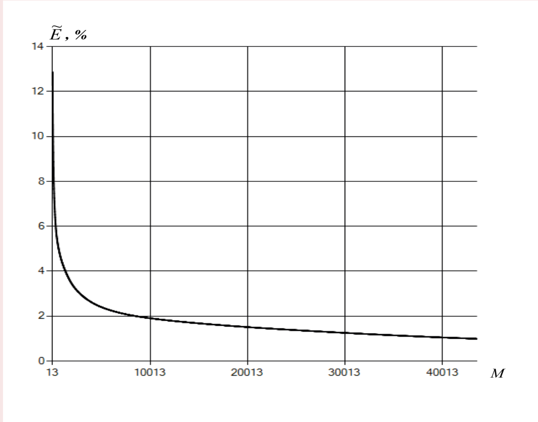
Source: own calculations.

Figure 10. Investment dynamics in the UR culture in 2000–2018 and their forecast in 2019–2023: budget (1), private (2)



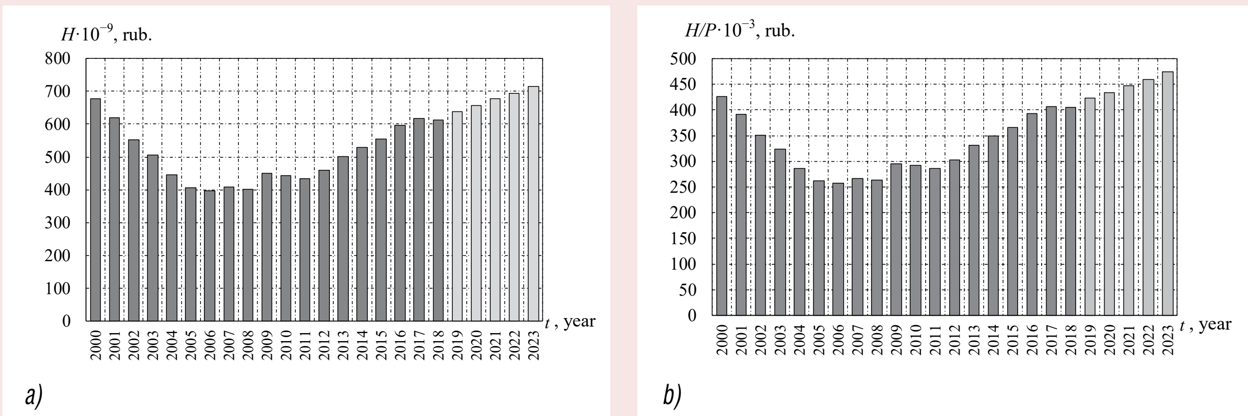
Source: own calculations.

Figure 11. Dependence of the neural network training quality indicator on the number of iterations



Source: own calculations.

Figure 12. Human capital dynamics (a) and specific value of UR human capital (b) for 2000–2018 and its forecast in comparable prices



Source: own calculations.

Figures 8–10 show the results of forecasting investments in the social sph*ere of UR for the period 2019–2023.* Figure 11 shows a graph of convergence to the 1% error level.

Deviation of pattern investment values from actual UR data for the period of 2000–2018 in health care is 1.4%, in education – 1.2%, in culture – 1.1%.

Based on the formula (10), the value of the HC UR for the period of 2000–2018 has been calculated. *Figure 12* shows the results of calculating the HC value for the period of 2000–2018 and the results of the problem solving of predicting the size and dynamics of the HC using the results of problem solving of predicting the volume of investments in the HC based on neural network modeling for the period of 2019–2023.

Conclusion

We have built a neural network algorithm, on the basis of which the forecasting human capital problem was solved on the example of the socio-economic system of the Udmurt Republic.

Neural network modeling of predicted values of investment volumes in human capital has shown its effectiveness. Thus, the deviation of the pattern values of investments in the components of the HC UR from the actual data for the period 2000–2018 amounted to 1.4%.

Calculations showed that the human capital value of the Udmurt Republic decreased in the 2000–2006 interval, and then there was an increase in this indicator. The studied indicator has been showing the highest growth rates since 2013, and its further growth is predicted. In 2018, the human capital value per one UR resident amounted to about 400 thousand rubles.

The results obtained qualitatively correlate with the dynamics of changes in the human development index of Russia, determined by the UN experts which in the period from 2000 to 2012 increased from 0.71 to 0.79, and subsequently, until 2017, remained almost constant to a value of 0.80.

The Udmurt Republic is a Russia's typical region in many socio-economic indicators[2]. It is characterized by the average Russian values of these indicators. Therefore, the results and conclusions obtained during the research can be extrapolated to the RF as a whole.

The proposed methodology for calculating and forecasting the value, structure and dynamics of human capital can be used in assessing and comparing the socio-economic situation of the regions of the Russian Federation.

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